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Diffusion of Polymer Additives in a Developing Turbulent Boundary Layer

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The results of an experimental study of the diffusion of water and aqueous polymer when tangentially injected from a line slot into a two-dimensional external turbulent boundary layer formed on a flat plate are presented. Concentration profiles were obtained by adding a tracer dye to the injected liquid and measuring the fluorescence, using a spectrophotometer, of samples taken from the flowfield. Empirical equations describing the transverse and longitudinal concentration field for given injections and flow parameters are presented. The results show that turbulent diffusion of polymer solution is suppressed compared to that of water when the concentration at the wall is higher than 0.75 wppm (weight parts per million) but increased below this value. The data also confirm the deduction of others that a series of regions can be defined, and that their length is dependent on the injection concentration and flow rate. A universal concentration profile is described for the final region which is in agreement with the observations of others.

Nomenclature

a = constant
c = polymer concentration, wppm
c_i = value of c at injection slot fluid, wppm
$\bar{c}$ = time average value of c at a point, wppm
$\bar{c}_m$ = wall value of $\bar{c}$, wppm
f = friction factor for polymer solution
Δf $= f - f_0$
G_i = volume flux of polymer solution per unit span, m^2/sec
h = slot thickness, mm
K_l $= u_i h c_i / [\lambda \int_0^{\delta/\lambda} \bar{u}\bar{c} d\xi]$ (correction factor)
n = velocity profile power law constant 7
Re_d = Reynolds number for tube flow based on the diameter
u,v = x,y components of the velocity vector, m/sec
$\bar{u},\bar{v}$ = time average value of u, v at a point, m/sec
$\bar{u}_{\max}$ = mean freestream velocity, m/sec
x,y = rectangular coordinates; x measured along the flow direction from the injection slot; y a normal distance from the surface, mm
δ = boundary-layer thickness, mm
$\bar{\delta}$ = mean boundary-layer thickness $= \frac{1}{x}\int_0^x \delta dx$, mm
λ = value of y at which $\bar{c}/\bar{c}_m = 0.5$, mm
ξ $= y/\lambda$

Subscripts

p = polymer
w = water

I. Introduction

THAT polymeric additives can reduce drag in turbulent flows has been well established by numerous authors. A widely accepted hypothesis of the mechanism involved in the drag reduction is that of the suppression of high-frequency turbulence in the buffer zone of the boundary layer. Consequently, for maximum drag reduction for a given polymer injection rate, it is essential to introduce and keep the additives efficiently in the wall region. A number of approaches may be used to introduce the additives into the crucial wall region. For example, homogeneous dispersion and injection or additive coatings may be used. In general, injection flows have more practical potential than other approaches.

The injected solution is diluted greatly by the turbulent mixing within the boundary layer; therefore, the drag reduction, which may be assumed to be dependent on the concentration in the wall region, is dependent on the diffusion rate of the additives in that region. The research reported in this paper is concerned with the subject of the prediction of the turbulent diffusion rate and concentration levels of injected drag-reducing polymer flows in a two-dimensional developing turbulent boundary layer.

II. Literature Review

Unfortunately, very few publications on the prediction of the turbulent diffusion rates of polymer solution have been reported in the open literature. Recent reviews by Gadd,[1] Hoyt,[2] and Palyvos[3] have covered the majority of publications relevant to drag reduction. However, a summary of the publications pertinent to polymer diffusion, including relevant Newtonian diffusion flows, is reviewed briefly. Poreh and Cermak[4] defined four zones in the downstream diffusion field for the diffusion of Newtonian fluids injected from a line slot into a two-dimensional developing turbulent boundary layer as follows: 1) an initial zone in which there are very large concentration gradients; 2) an intermediate zone in which the rate of growth of the diffusion boundary-layer thickness is higher than that of the hydrodynamic boundary-layer thickness, and the diffusion boundary layer is submerged in the hydrodynamic boundary layer; 3) a transient zone in which the main stream starts to influence the growth of the diffusion layer; and 4) a final zone in which diffusion of matter beyond the hydrodynamic boundary layer into the main fluid is controlled by turbulent fluctuation in the main fluid.

Fabula and Burns[5] studied the turbulent diffusion when tangentially injecting water and a constant concentration of drag-reducing polymer solution from a line slot into a developing, two-dimensional boundary layer. They observed good agreement between the concentration profiles for Newtonian fluids and diluted polymer solutions. Wu,[8] using a laser-phototransistor, measured the concentration profiles at one location in the final zone for various concentrations of polymer when injected from a line slot over a flat plate. His

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results indicate a suppressed turbulent diffusion of polymer solution, but he did not obtain any general correlations for the concentrations. Poreh et al.[9,10] analyzed the diffusion of diluted drag-reducing polymers and suggested that the diffusion rate and the drag are reduced when polymer injection is used. However, the reduction in the diffusion rate is relatively small, and polymers ejected near the wall diffuse over the entire length within a distance of 100 $\bar{\delta}$. The turbulent diffusion of polymer in the final zone was studied by Wetzel and Ripken,[6] who showed that the concentration profiles could be correlated using a relationship developed by Morkovin[7] for gases. At present, there appear to be no available papers published on empirical data covering the four regions as described in the theoretical papers by Poreh and Hsu.[9,10]

III. Experimental Apparatus

The experiments were performed in a water tunnel that could be operated in an open- or closed-circuit mode, and had a 1.3-m-long × 0.152 m-i.d. plexiglass test section. A horizontal flat plate, 1.3 m long × 0.152 m wide with a 12.7-mm thickness, was centrally located in the test section with its large surfaces horizontal. A 76-mm-wide × 0.64-mm-deep transverse injection slot (see Fig. 1), located on the upper face of the flat plate 137 mm downstream from the leading edge, was used to inject the solutions tangentially into the boundary layer. The injected solution was supplied to both ends of a transverse chamber situated behind the injection slot to obtain a uniform injection velocity profile. Mean concentration profiles were obtained at five sampling stations 50, 146, 304, 558, and 884 mm downstream from the injection slot which covered the four diffusion zones for polymer injection flows. The measurements were made using 1-mm-o.d. × 0.5-mm-i.d. right-angle total head sampling probes. The samples were collected at only one longitudinal location each time to avoid the effects of disturbances created behind each probe. The flow rate into the sampling tubes was sufficiently small that no disturbance in the main flow would occur. The concentration profiles were obtained by adding a tracer dye (Rhodamine WT) to the injected fluid and measuring the fluorescence, using a spectrophotometer (Gilford type, model 2400), of samples taken from the flowfield. The accuracy of the measurements of the concentration of the tracer fluid was about 5 ppb.

Mean velocity profiles were obtained upstream from the plate and at three locations 95, 539, and 984 mm downstream from the injection slot using pitot tubes for a constant $\bar{u}_{\max}$ of approximately 6 m/sec and for ratios of $\bar{u}_{\max}/u_i$ in the range 6 to 40. The additive used was Reten 423 (obtained from Hercules Inc.), which is an anionic acrylic polymer with an assessed molecular weight of between 10^7 to 10^8 and is known to have high resistance to shear degradation. A typical drag reduction curve for the polymer in tube flow is given in Fig. 2, showing that drag is reduced with concentrations of up to 150 wppm and increased above this value. Furthermore, the maximum effectiveness occurs with about 10 wppm. Similar results can be obtained for external flows, but they are less convenient to obtain than to tube flows because of the number of parameters involved. The aqueous polymer solution was mixed about 24 hr before use and then stored in a pressure vessel. Compressed air supplied to the top of the vessel was used to force the solution through the injection slot.

IV. Experimental Results

Figure 3 shows dimensionless concentration profiles of $\bar{c}/c_i$ vs y at five locations downstream from the injection slot for water and polymer solution injections at different injection flow rates and concentrations. A significant difference in the concentration profiles between water and high-concentration polymer injection is noticeable, indicating that the injected polymer will be retained near the wall. The following observations also may be made.

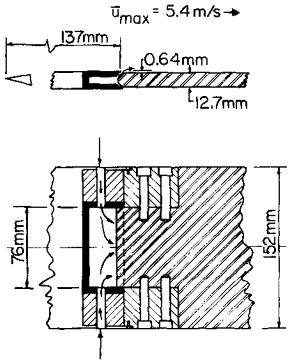


Fig. 1 Sectional view of injection plenum.

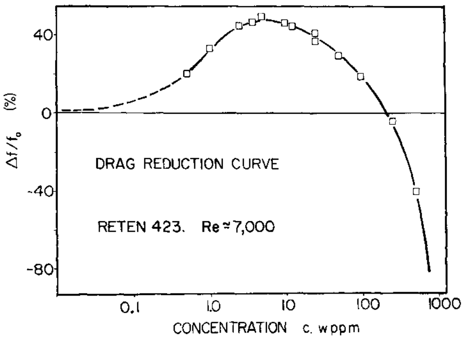


Fig. 2 A typical drag reduction curve for Reten 423.

A. Water Injection

In all cases, the concentration profiles were similar (see Fig. 3a), indicating that all of the stations were within the final zone, and the first three zones were upstream of the first station, which is not in agreement with Poreh's predictions. The mean concentration profiles can be described by a dimensionless universal curve, as shown in Fig. 4, of the following form

$$\bar{c}/\bar{c}_m = f(\xi) \tag{1}$$

The function $f(\xi)$ appears to be independent of the location and can be described by the following equation, which was presented first by Morkovin[7]

$$\bar{c}/\bar{c}_m = \exp[-0.693\ (y/\lambda)^{2.15}] \tag{2}$$

The data for λ as a function of x have been correlated using the empirical equations

$$\lambda = 0.284\ (x)^{0.47} \tag{3}$$

As shown in Fig. 5, it therefore appears that λ is independent of injection flow rate, if the equation is true, for a wide range of injection flows.

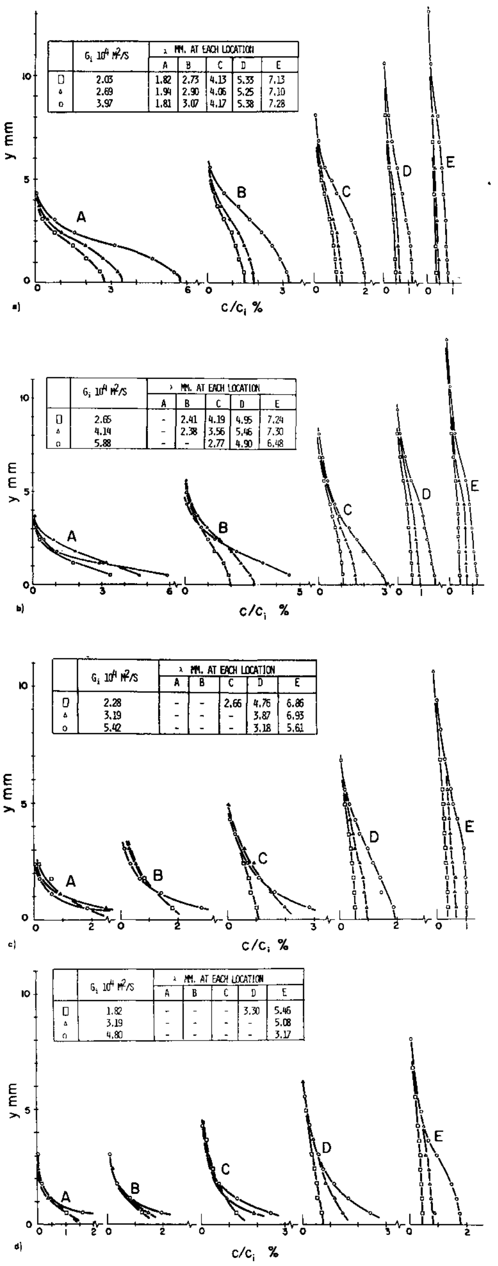
The variation of the dimensionless wall concentration can be approximated by the following empirical equations, as shown in Fig. 6

$$\bar{c}_m \bar{u}_{\max}/c_i u_i = 25.63\ (x/h)^{-0.834} \tag{4}$$

where $h = 0.64$ mm for the present data, and λ/δ approaches a constant value of 0.45, as shown in Fig. 7 within the test section length, i.e., $12.5 < x/\delta < 82$, which is the condition of the final zone described by Poreh.[4]

B. Polymer Solution Injection

The concentration profiles can be divided into at least two groups. The first group represents the initial zone, in which



a)

	G_i 10^4 M^2/S	λ MM. AT EACH LOCATION				
		A	B	C	D	E
□	2.03	1.82	2.73	4.13	5.33	7.13
△	2.69	1.94	2.90	4.06	5.25	7.10
○	3.97	1.81	3.07	4.17	5.38	7.28

b)

	G_i 10^4 M^2/S	λ MM. AT EACH LOCATION				
		A	B	C	D	E
□	2.65	-	2.41	4.19	4.95	7.24
△	4.14	-	2.38	3.56	5.46	7.30
○	5.88	-	-	2.77	4.90	6.48

c)

	G_i 10^4 M^2/S	λ MM. AT EACH LOCATION				
		A	B	C	D	E
□	2.28	-	-	2.66	4.76	6.86
△	3.19	-	-	-	3.87	6.93
○	5.42	-	-	-	3.18	5.61

d)

	G_i 10^4 M^2/S	λ MM. AT EACH LOCATION				
		A	B	C	D	E
□	1.82	-	-	-	3.30	5.46
△	3.19	-	-	-	-	5.08
○	4.80	-	-	-	-	3.17

Fig. 3a) Concentration profiles ($c_i = 0$ wppm). b) Concentration profiles ($c_i = 100$ wppm). c) Concentration profiles ($c_i = 250$ wppm). d) Concentration profiles ($c_i = 500$ wppm).

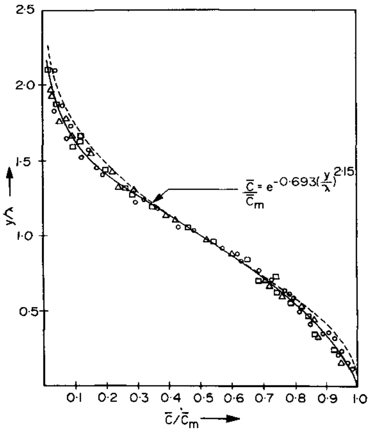


Fig. 4 Universal concentration profile (water injection; all data, and $c_i = 0$ wppm).

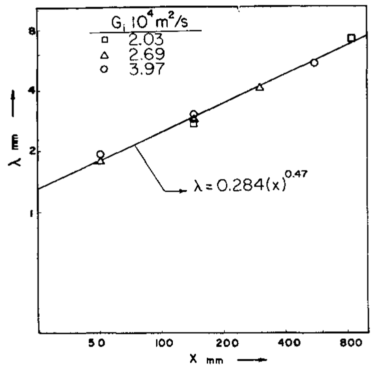


Fig. 5 λ vs x (water injection).

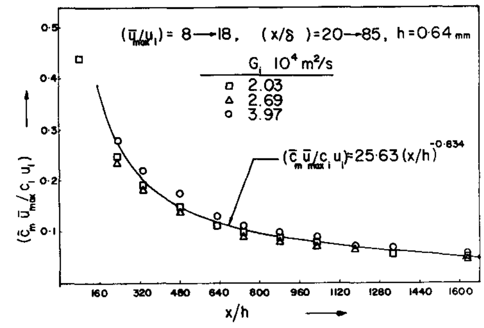


Fig. 6 $\bar{c}_m \bar{u}_{max}/c_i u_i$ vs x/h **(water injection).**

the concentration gradient is very steep, and a universal curve could not be obtained, since it was impossible to obtain an accurate value for $\bar{c}_m$. However, the data of the second group, which represent the intermediate and final zones, there being no indication of a transient zone, where correlated with good accuracy using the following universal equation, as shown in Fig. 8

$$\bar{c}/\bar{c}_m = \exp[-0.693\ (y/\lambda)^a] \tag{5}$$

The constant a is dependent on the location downstream from the injection slot, the velocity ratio $(\bar{u}_{max}/u_i)$, the injection concentration, and the slot height; that is

$$a = f\ (\bar{u}_{max}/u_i, x/h,\ c_i)$$

where a is 1.7 for the intermediate zone and 2.15 for the final zone.

Figure 9 shows the variation of (λ/δ) as a function of $u_i h\ c_i/\bar{u}_{max}x$ for various injection concentrations and velocities for the final zone. The data correlated accurately using the following equation

$$\lambda/\delta = 0.53 - 0.18 \times 10^8\ (u_i\ h\ c_i/\bar{u}_{max}x) \tag{6}$$

for the conditions

$$x/\delta \geq 50,\ u_i\ h\ c_i/\bar{u}_{max}x \geq 10^{-8}$$

The wall concentration also can be obtained from Fig. 9 and is given by the following empirical equation

$$\bar{c}_m = 0.81 \times 10^8\ (u_i\ h\ c_i/\bar{u}_{max}x) \tag{7}$$

for

$$u_i\ h\ c_i/\bar{u}_{max}x \leq 3.5 \times 10^{-8}$$

It is apparent from Fig. 9 that λ increases to a maximum and then decreases as $u_i\ h\ c_i/\bar{u}_{max}x$ increases, thus indicating that

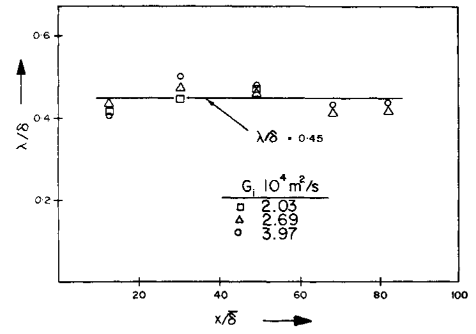


Fig. 7 λ/δ vs x/δ (water injection).

the diffusion rate for low polymer concentration is higher than that for water for the range $0<u_i hc_i/\bar{u}_{\max} x<1.25\times 10^{-8}$ or $\bar{c}_m$ in the range 0 to 0.75, and will be less than that for water for $u_i hc_i/\bar{u}_{\max} x>1.25\times 10^{-8}$ or $\bar{c}_m>0.75$.

V. Analysis and Discussion

A. Water Injection

There were no obvious distinct zones in the longitudinal range used, and a universal concentration profile representing the final zone was obtained for all of the water data. This is contrary to what might be predicted from Poreh's[4] work, which would suggest four distinct zones within the same empirical length range as that used in this work. However, this difference may be expected, since the turbulence intensity, slot geometry will affect the diffusion rate considerably and must be considered if critical comparisons are to be made.

B. Polymer Solution Injection

The initial, intermediate, and final diffusion zones were covered for the longitudinal range used for the polymer experiments, with no obvious transition zones discernible. It is apparent that, by increasing the quantity of the polymer injected, the initial zone will be extended greatly compared to the intermediate and the transient zones. This is mainly because the bulk of the additives will be submerged in the viscous sublayer, where the turbulence is very low, and the molecular diffusion, which is very small, will be the primary diffusion mechanism. Thus the overall effect of polymer injection is to increase the initial zone length while moving the start of the intermediate and final zones further downstream, and consequently appreciably affecting the distribution of $\bar{c}_m$ with respect to x.

C. Continuity Equation

It was found that the mean velocity profiles for the hydrodynamic boundary layer can be represented accurately by a standard 1/7th power law of the form

$$\bar{u}/\bar{u}_{\max} = (y/\delta)^{1/7} \tag{8}$$

and may be used for polymer solutions, assuming that the velocity profiles will be approximately the same for water and polymer solution injection, when $\bar{c}_m$ is small.

The conservation of mass for the two-dimensional flow can be written as follows.

$$\int_0^\infty cu\, dy = u_i h\, c_i \tag{9}$$

By introducing Eqs. (1) and (8) into Eq. (9), the following relationship can be outlined

$$\frac{u_i\, h\, c_i}{\bar{u}_{\max}\, \lambda \bar{c}_m} = \left(\frac{\lambda}{\delta}\right)^{1/7} \int_0^{\delta/\lambda} (\xi)^{1/7} f(\xi)\, d\xi + \int_{\delta/\lambda}^{\infty} f(\xi)\, d\xi \tag{10}$$

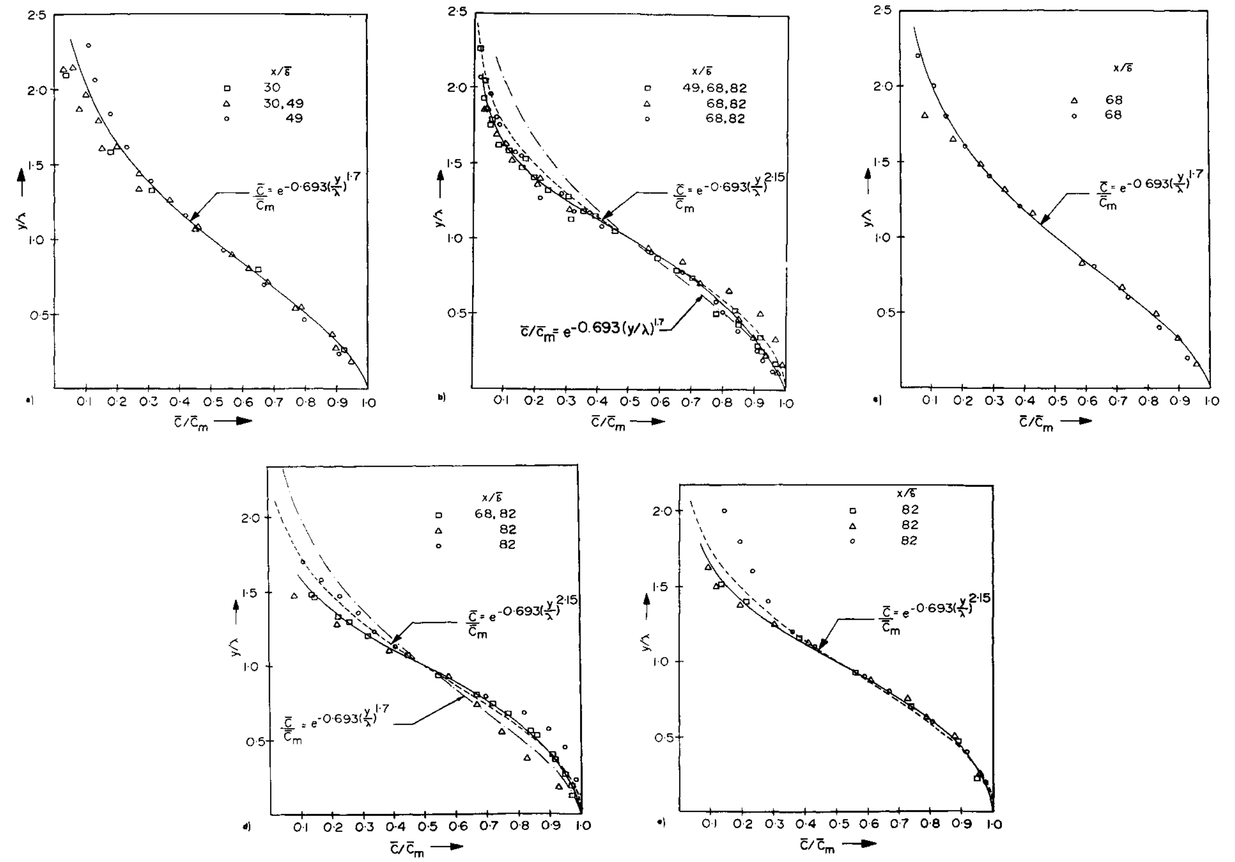


Fig. 8 a) Universal concentration profile ($c_i=100$ wppm; intermediate zone. b) Universal concentration profile ($c_i = = 100$ wppm; final zone. c) Universal concentration profile ($c_i=250$ wppm; intermediate zone). d) Universal concentration profile ($c_i=250$ wppm; final zone. e) Universal concentration profile ($c_i=500$ wppm; final zone).

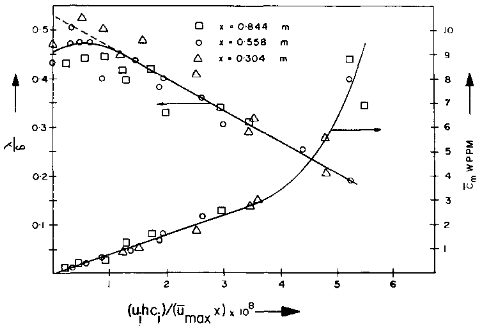


Fig. 9 λ/δ and $\bar{c}_m$ vs $u_i\ h\ c_i/\bar{u}_{\max}x$.

Since from the empirical data $\lambda/\delta \leq 0.5$, then $f(\xi) \to 0$ in the range δ/λ to ∞; therefore

$$\int_{\delta/\lambda}^{\infty} f(\xi)\ d\xi \to 0$$

Furthermore, in the final zone δ/λ approaches a constant value, and consequently

$$\int_{0}^{\delta/\lambda} (\xi)^{1/7} f(\xi)\ d\xi \to \text{const value}$$

Therefore, the continuity equation for the final zone may be reduced to

$$\frac{u_i\ h\ c_i}{\bar{u}_{\max}\ \lambda \bar{c}_m} = \text{const}\ k_l \tag{11}$$

In actual fact, the experimental data showed that k_l varies from 0.9 to 1.8 as x is increased, and therefore the continuity equation must be modified by introducing the empirical coefficient K_l, that is

$$\frac{u_i\ h\ c_i}{\bar{u}_{\max}\ \lambda \bar{c}_m} = K_l \left(\frac{\lambda}{\delta}\right)^{1/7} \int_{0}^{\delta/\lambda} (\xi)^{1/7} f(\xi)\ d\xi \tag{12}$$

where $K_l = 0.31\ (x)^{0.26}$ for the present data.

This means that the flow is either three-dimensional or Eq. (2) is not valid for high values of ξ. However, measurements of the mean concentration in the transverse direction indicated that the flowfield was very nearly two-dimensional. It also is apparent that Eq. (2) is not valid for high values of ξ, which means that the injected fluid will diffuse beyond the hydrodynamic boundary layer in the final zone.

It is generally accepted that for external flows drag reduction will occur only if $\bar{c}_m$ is within a certain range dependent on the additive, whereas the drag increases above this range. For Reten 423, this range is about 0.1 to 150 wppm. As shown in the present work, $\bar{c}_m$ varies considerably with the distance from the injection slot, and consequently it is difficult to achieve drag reduction over the majority of a surface without incurring drag increase over the initial zone. Therefore, it is anticipated that in most practical cases a series of injection slots will be required if drag reduction is to be maximized. It is apparent that slot geometry, type of additive, and injection flow rate and concentration must be considered when optimizing a system design for drag reduction. Therefore, more data on diffusion phenomena, injection methods, and drag reduction are necessary if efficient designs are to be achieved.

VI. Conclusions

From the results, the following general conclusions can be drawn:

1) The diffusion fields for polymer solution and Newtonian fluid injections are in some ways similar and can be divided into zones, which is in some agreement with the work of Poreh et al.,[4] Fabula et al.,[5] and Wetzel et al.[6]

2) A comparison between water and polymer solution injections shows that for water injection the length of the first two zones is relatively short and insignificant, whereas for polymer injection the lengths of the first two zones and particularly the initial zones are very significant.

3) Within the range of the flow parameters for the present research, the diffusion rate of very dilute drag-reducing aqueous polymer solutions is increased for very low wall concentration, that is, when $0 < \bar{c}_m < 0.75$ wppm; however, diffusion is suppressed when $\bar{c}_m > 0.75$ wppm.

4) A general equation of the form of Eq. (2) can be used to predict the concentration profile for water and polymer solution injections and can be used in conjunction with the empirical Eq. (3) or (7) to obtain a reasonably accurate prediction of the concentration distribution in the final zone with respect to the injection conditions.

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